

Unfolding the energy spectrum

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Parts of the project:

- 1) Modelling the response of the detector: building the matrix.
- 2) Unfolding the spectrum: solving the linear system.

1) Modelling the response of the detector

A probability density $f(x)$ of a variable x is measured by our detector. The detector has finite resolution and smearing and acceptances losses. What we measure is

$$f'(x') = \int_{-\infty}^{\infty} t(x', x) f(x) dx$$

where $t(x', x)$ is the transfer function.

In order to determine $t(x', x)$, we use Monte Carlo simulations.

$$g'(x') = \int_{-\infty}^{\infty} t(x', x) g(x) dx$$

Usually one combines events in bins. Thus

$$d'_\mu = \sum_\nu T_{\mu\nu} d_\nu \quad m'_\mu = \sum_\nu T_{\mu\nu} m_\nu$$

where

$$\begin{aligned} T_{\mu\nu} &= \frac{\int_{\mu} dx' \int_{\nu} dx t(x', x) f(x)}{\int_{\nu} dx f(x)} \\ &\approx \frac{\int_{\mu} dx' \int_{\nu} dx t(x', x) g(x)}{\int_{\nu} dx g(x)} \end{aligned}$$

An estimate of $T_{\mu\nu}$ is

$$T_{\mu\nu} \approx \frac{m'_{\mu\nu}}{m_{\nu}}$$

Problems:

The Monte Carlo simulations we have include simulation of the electronics which depends on the beam geometry.

2) Unfolding

- **Unfolding by matrix inversion**

$$Td = d' \quad \Rightarrow \quad d = (T^t T)^{-1} T^t d'$$

If the transfer matrix is quadratic this simplifies to

$$d = T^{-1} d'$$

However because of statistical uncertainties in the bin content and roundoff errors, the solution is a useless strong oscillating distribution.

- **Least square and maximum likelihood method**

The contents of the bins of the unfolded distribution are considered as free parameters in a fit.

In order to avoid oscillations, a regularization term is added to the purely statistical χ^2 ,

$$\chi^2 = \chi_{\text{stat}}^2 + R_{\text{regu}}$$

such that

$$R_{\text{regu}} \propto \left(\frac{\partial^2 f}{\partial x^2} \right)^2$$

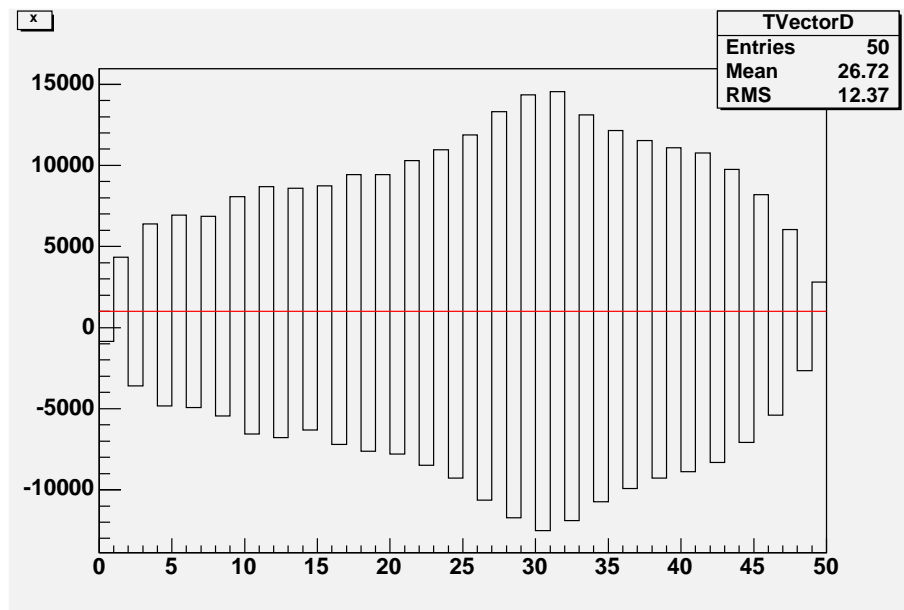
SVD decomposition is used to solve the remaining linear system and to get the optimal value of the weight of the regularization term.

$$A = USV^t$$

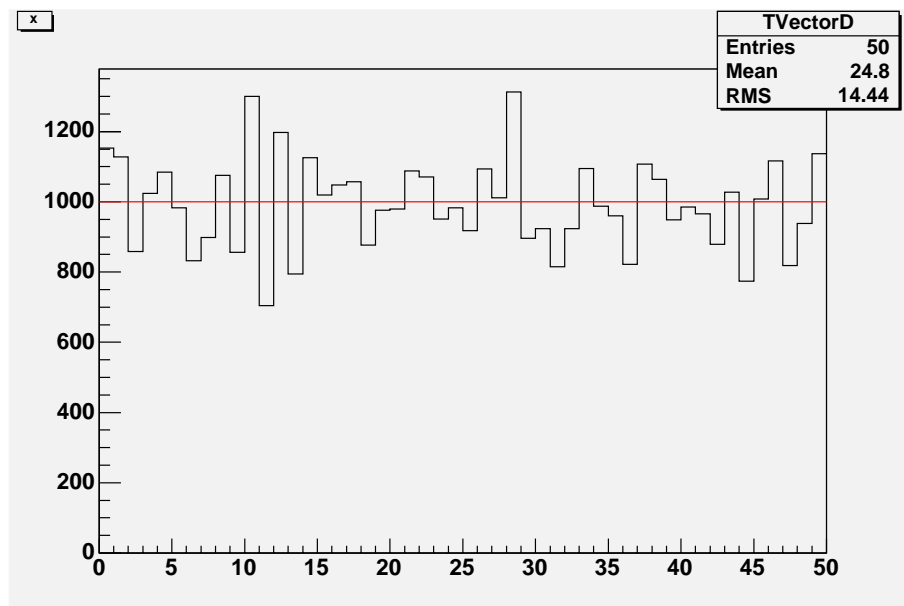
The whole algorithm is described in:

A. Hocker and V. Kart, "SVD approach to data unfolding", Nucl. Instr. and Methods A 372 (1996) 469-481.

Undamped distribution



Damped distribution



State of the project:

- The unfolding procedure is already implemented and seems to work.
- I am still working on finding the matrix elements of the detector from the Monte Carlo simulations we have.
- A clear distinction needs to be made in the reconstruction process between the electronics simulation and the actual events simulation, which is not done, in order to understand the detector response. Work is needed in this direction.